

St Aidan's R.C First School
Progression in Calculations Policy

Autumn 2014

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Introduction

Written methods of calculations are based on mental strategies. Each of the four operations builds on mental skills which provide the foundation for jottings and informal written methods of recording. Skills need to be taught, practised and reviewed constantly. These skills need to be secure before leading on to more formal written methods of calculation.

Strategies for calculation need to be supported by familiar models and images to reinforce understanding. When teaching a new strategy it is important to start with numbers that the child can easily manipulate so that they can understand the concept.

The transition between stages should not be hurried as not all children will be ready to move on to the next stage at the same time, therefore the progression in this document is outlined in stages. Previous stages may need to be revisited to consolidate understanding when introducing a new strategy. A sound understanding of the number system is essential for children to carry out calculations efficiently and accurately.

Progression in Teaching Addition

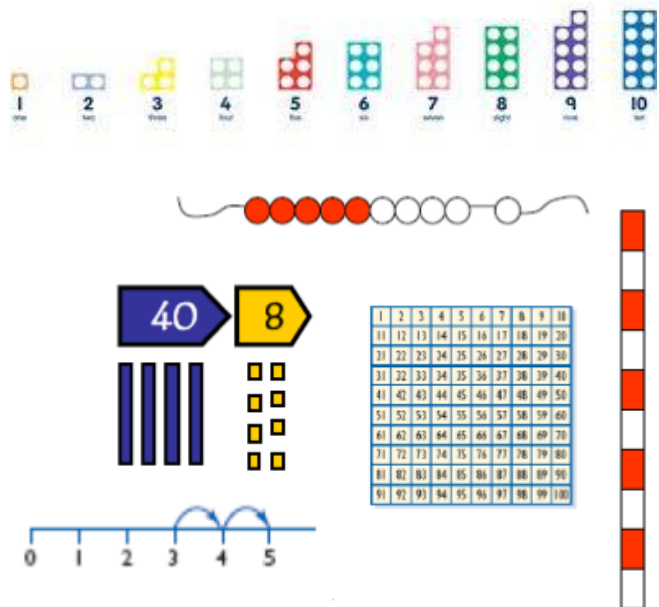
Mental Skills

Recognise the size and position of numbers
 Count on in ones and tens
 Know number bonds 5,6,7,8,9,10 and 20
 Double numbers to 10+10
 Add multiples of 10 to any number
 Partition and recombine numbers
 Know the inverse of + is - check!



Models and Images

Counting apparatus
 Place value apparatus
 Place value cards
 Number tracks
 Numbered number lines
 Marked but unnumbered number lines
 Numicon
 Empty number lines
 Hundred square
 Counting stick
 Bead string
 Models and Images charts
 ITPs

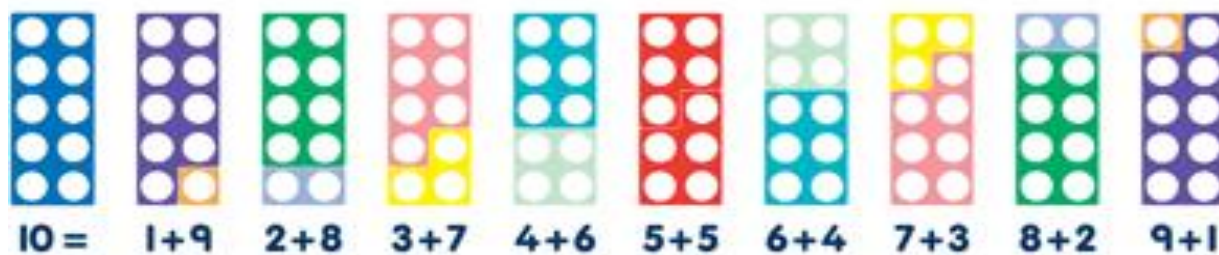


ITPs–Number Facts, Ordering Numbers, Number Grid, Counting on and back in ones and tens

Key Vocabulary

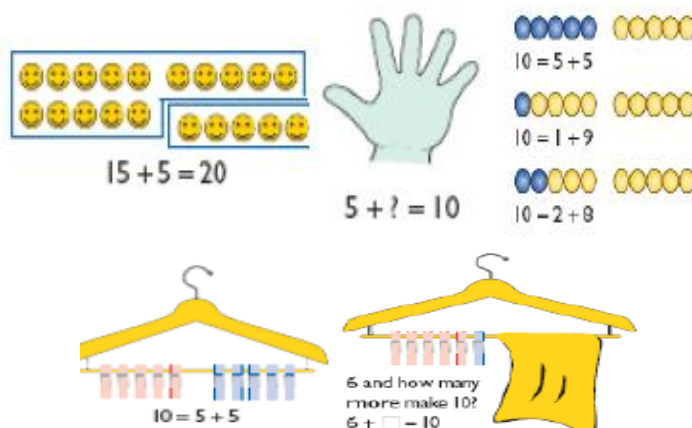
Add/Addition
 Double
 And
 Count on
 More
 Sum (find the sum of...)
 Total
 Altogether
 Increase

1. Recognise numbers 0 to 10	<div>012345678910</div> <div><div>1one</div><div>2two</div><div>3three</div><div>4four</div><div>5five</div><div>6six</div><div>7seven</div><div>8eight</div><div>9nine</div><div>10ten</div></div>																																																																																																				
2. Count reliably up to 10 everyday objects	<div><div><div><div></div><div></div><div></div><div></div><div></div><div></div></div><div>1, 2, 3, 4, 5, 6 ... there are 6 teddies</div></div></div>																																																																																																				
3. Find one more than a number	<div><div>1two</div><div>2two</div><div>3three</div><div>4four</div><div>5five</div><div>6six</div><div>7seven</div><div>8eight</div><div>9nine</div><div>10ten</div></div>																																																																																																				
4. Count in ones and tens	<div><div><div><div></div><div></div><div></div><div></div><div></div><div></div></div><div>25 + 10 = 35</div></div><div><table><tr><td>1</td><td>2</td><td>3</td><td>4</td><td>5</td><td>6</td><td>7</td><td>8</td><td>9</td><td>10</td></tr><tr><td>11</td><td>12</td><td>13</td><td>14</td><td>15</td><td>16</td><td>17</td><td>18</td><td>19</td><td>20</td></tr><tr><td>21</td><td>22</td><td>23</td><td>24</td><td>25</td><td>26</td><td>27</td><td>28</td><td>29</td><td>30</td></tr><tr><td>31</td><td>32</td><td>33</td><td>34</td><td>35</td><td>36</td><td>37</td><td>38</td><td>39</td><td>40</td></tr><tr><td>41</td><td>42</td><td>43</td><td>44</td><td>45</td><td>46</td><td>47</td><td>48</td><td>49</td><td>50</td></tr><tr><td>51</td><td>52</td><td>53</td><td>54</td><td>55</td><td>56</td><td>57</td><td>58</td><td>59</td><td>60</td></tr><tr><td>61</td><td>62</td><td>63</td><td>64</td><td>65</td><td>66</td><td>67</td><td>68</td><td>69</td><td>70</td></tr><tr><td>71</td><td>72</td><td>73</td><td>74</td><td>75</td><td>76</td><td>77</td><td>78</td><td>79</td><td>80</td></tr><tr><td>81</td><td>82</td><td>83</td><td>84</td><td>85</td><td>86</td><td>87</td><td>88</td><td>89</td><td>90</td></tr><tr><td>91</td><td>92</td><td>93</td><td>94</td><td>95</td><td>96</td><td>97</td><td>98</td><td>99</td><td>100</td></tr></table></div></div>	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36	37	38	39	40	41	42	43	44	45	46	47	48	49	50	51	52	53	54	55	56	57	58	59	60	61	62	63	64	65	66	67	68	69	70	71	72	73	74	75	76	77	78	79	80	81	82	83	84	85	86	87	88	89	90	91	92	93	94	95	96	97	98	99	100
1	2	3	4	5	6	7	8	9	10																																																																																												
11	12	13	14	15	16	17	18	19	20																																																																																												
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91	92	93	94	95	96	97	98	99	100																																																																																												
5. Begin to relate addition to combining two groups of objects	<div><div><div><div></div><div></div><div></div></div><div>and</div><div><div></div><div></div></div><div>makes 5</div></div></div>																																																																																																				
6. Count along a number line to add numbers together	<div><div>3 + 2 = 5</div><div><div></div><div></div><div></div><div></div><div></div><div></div></div></div>																																																																																																				
7. Begin to use the + and = signs to record mental calculation in a number sentence	<div><div>6 + 4 = 10</div></div>																																																																																																				
8. Know doubles of numbers and their halves	<div><div>5 + 5 = 10 10 - 5 = 5</div><div><div></div><div></div><div></div><div></div><div></div><div></div></div></div>																																																																																																				

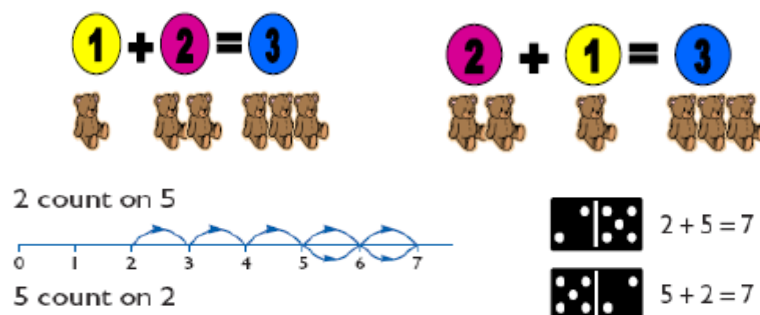


Numicon is a key teaching tool in the teaching of addition.

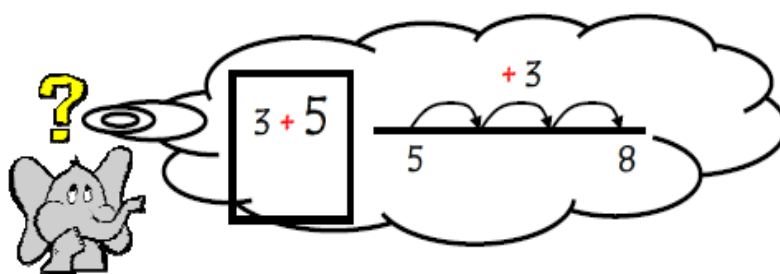
9. Know by heart all pairs of numbers with a total of 5,6,7,8,9,10 and 20



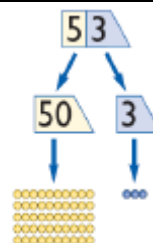
10. Know that addition can be done in any order, but that it is easier from the larger – check!



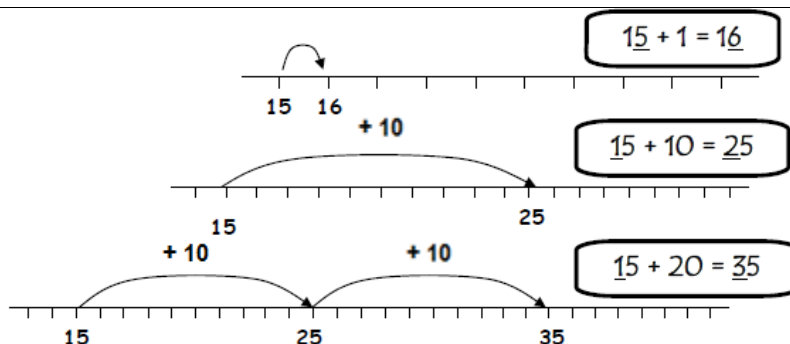
11. Put the biggest number first and count on



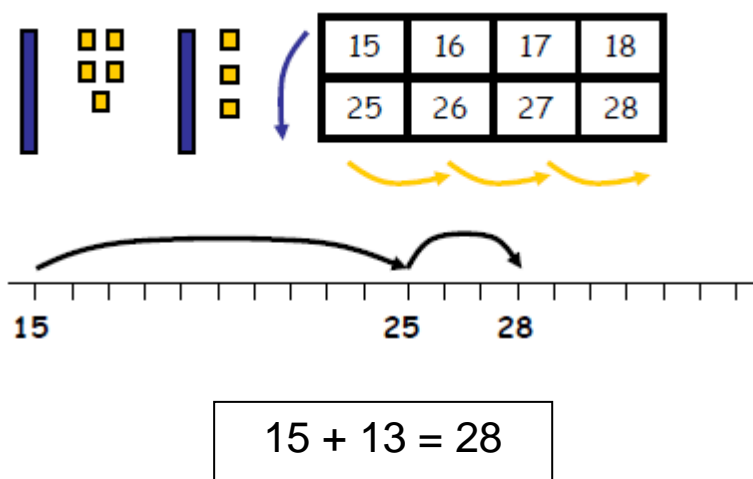
12. Begin to partition numbers in order to add



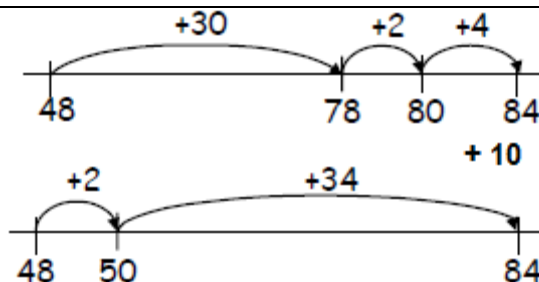
13. Know which digit changes when adding 1s or 10s to any number



14. Adding two two-digit numbers
Counting in tens and ones
Partitioning and recombining

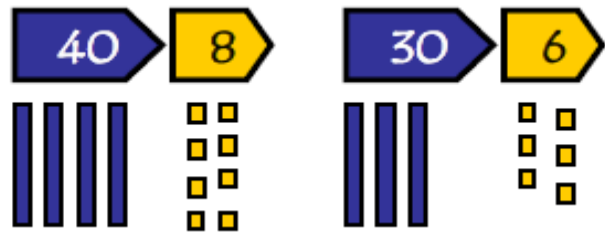


15. Adding two two-digit numbers;
Using a number line



OR

Using place value cards and place value apparatus (Dienes) to partition numbers and recombine to include exchanging units for tens (using dienes)



$$\underline{48 + 36}$$

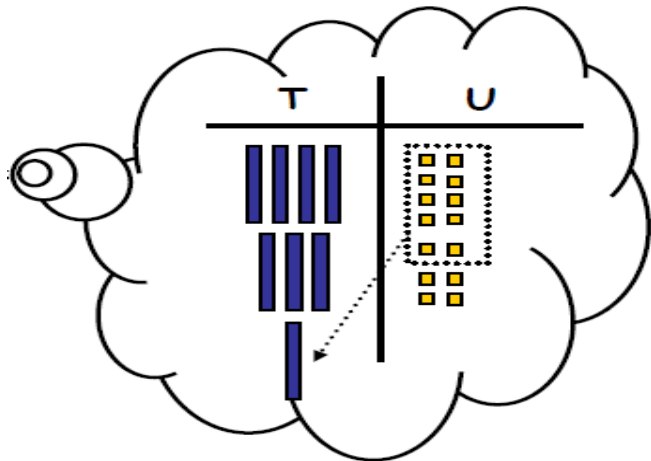
$$40 + 30 = 70$$

$$8 + 6 = 14$$

$$70 + 14 = 84$$

16. Expanded method

It is important that the children have a good understanding of place value and partitioning using concrete resources and visual images to support calculations. The expanded method enables children to see what happens to numbers in the standard written method



$$48 + 36$$

$$48$$

$$+ \underline{36}$$

$$14$$

$$\underline{70}$$

$$\underline{84}$$

(Add units first)

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17. Standard written method

The previous stages reinforce what happens to the numbers when they are added together using more formal written methods.

10

$$\begin{array}{r} 48 \\ + 36 \\ \hline 84 \end{array}$$

(‘exchange ten’ sitting on the line)

Progression in Teaching Subtraction

Mental Skills

Recognise the size and position of numbers

Count back in ones and tens

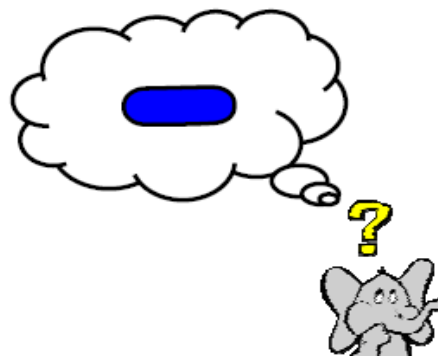
Know number facts for all numbers to 20

Halve all numbers to 20

Subtract multiples of 10 from any number

Partition and recombine numbers (only partition the number to be subtracted)

Know the inverse of $-$ is $+$ check!



Models and Images

Counting apparatus

Place value apparatus

Place value cards

Number tracks

Numbered number lines

Numicon

Marked but unnumbered lines

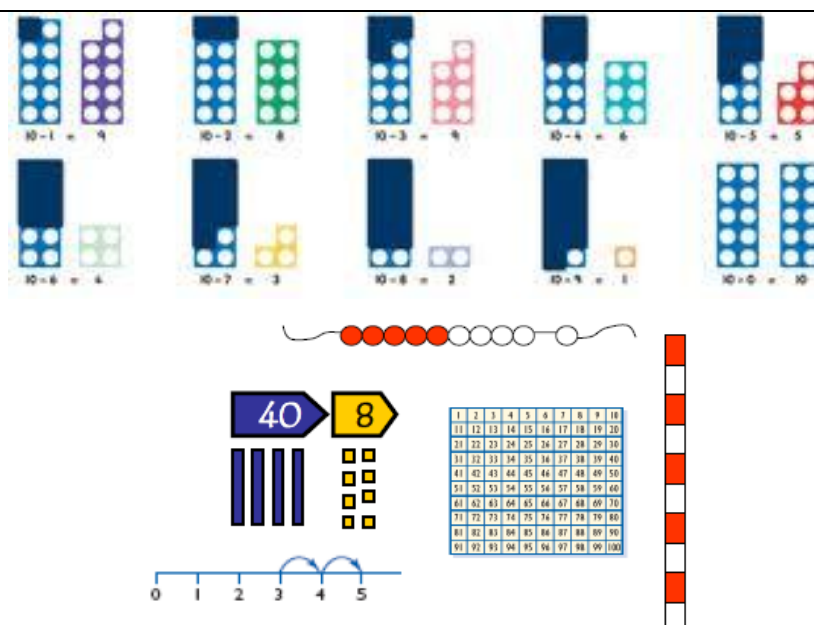
Hundred square

Empty number lines

Counting stick

Bead strings

Models and Images Charts



ITPs– Number Facts, Counting on and back in ones and tens, difference

Key Vocabulary

Subtract

take away

minus

count back

less

fewer

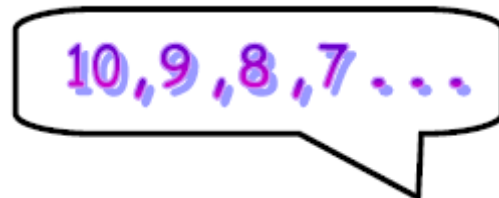
difference between

half

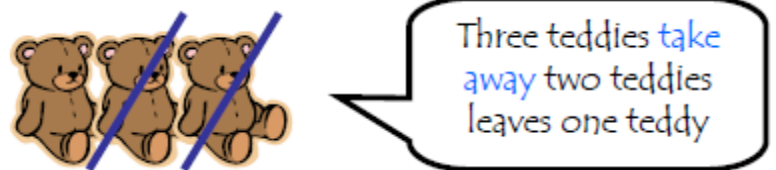
1.Begin to count backwards in familiar contexts such as number rhymes or stories



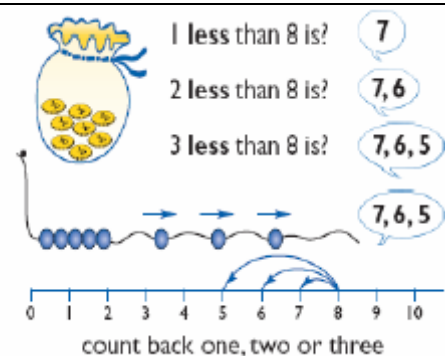
2. Continue the count back in ones from any given number



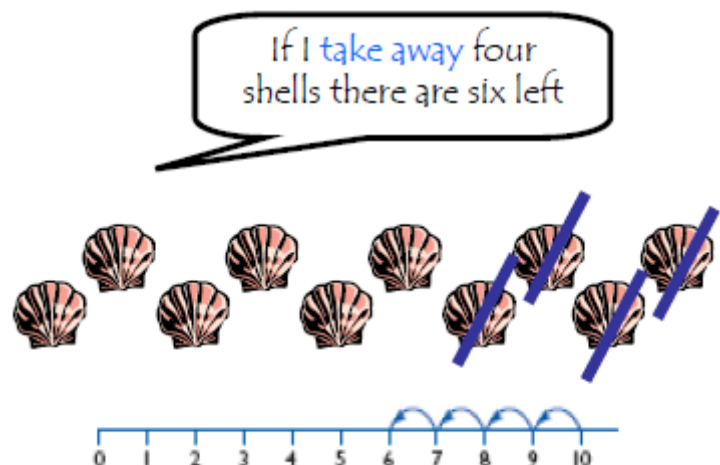
3.Begin to relate subtraction to 'taking away'



4.Find one less than a number

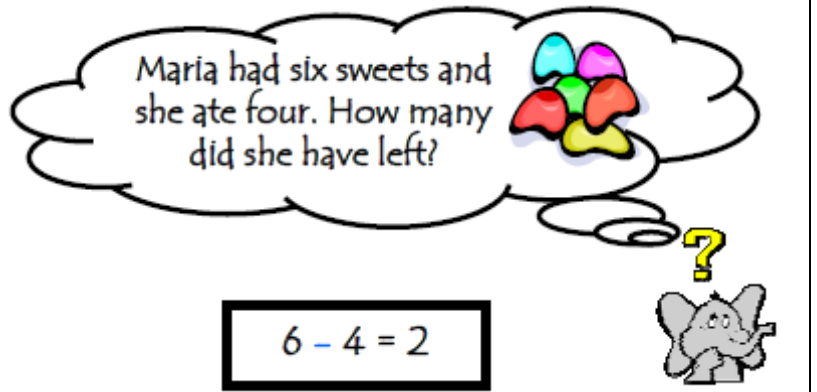


6.Count backwards along a number line to take away when numbers are further apart e.g. 10-2 (10 in your head 9,8)




7.Begin to use the – and = signs to record mental calculations in a number sentence

Maria had six sweets and she ate four. How many did she have left?







$$6 - 4 = 2$$

8.Know by heart subtraction facts for numbers up to 10 and 20




$$6 + ? = 10 \quad ? + 6 = 10$$

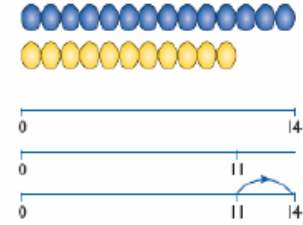
$$10 - 6 = ? \quad 10 - 4 = 6$$

		$20 = 12 + 8$	$8 + 12 = 20$
		$20 - 8 = 12$	$20 - 12 = 8$

9.Begin to find the difference by counting up from the smallest number when numbers are closer together e.g. 10-8 (8 in your head 9,10 and the answer is 2).



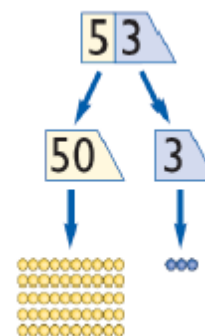
The difference is?



The difference between 11 and 14 is 3.
 $14 - 11 = 3$
 $11 + \square = 14$

Find the difference/how many more in relation to Bar Charts

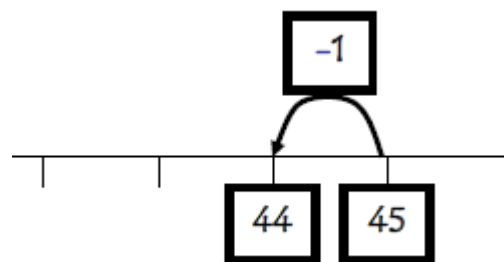
10.Begin to partition numbers in order to take away



11. Subtract 1 from a two – digit number

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

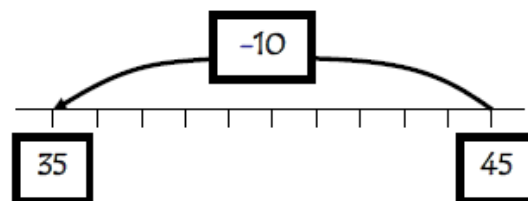
$$45 - 1$$



12. Subtract 10 from a two-digit number

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

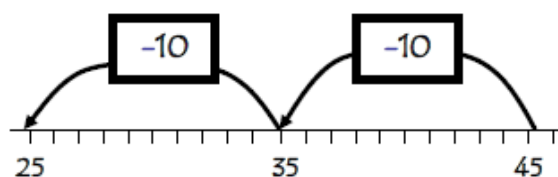
$$45 - 10$$



13. Subtract multiples of 10 from any number

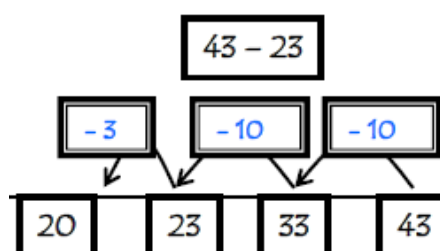
1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

$$45 - 20$$



(Introduce number line not starting at 0)

14. Partition the number to be subtracted (no exchanging)

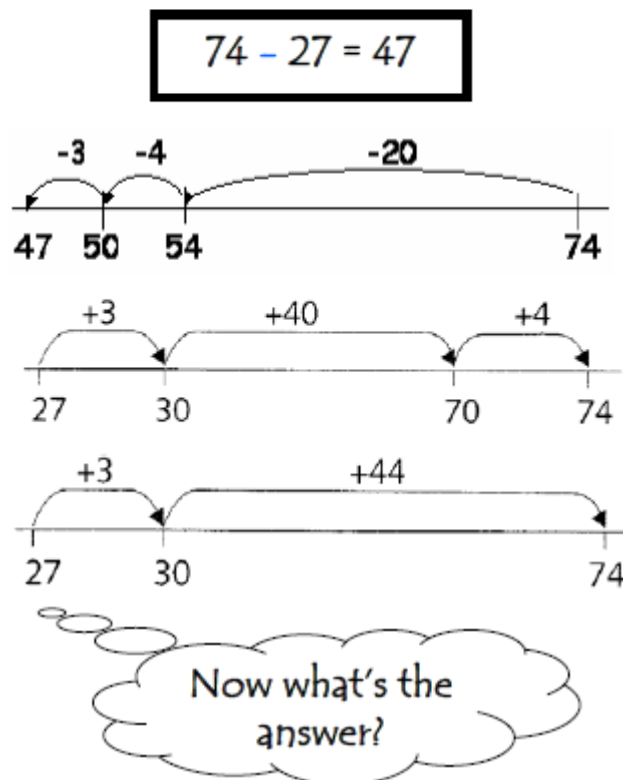


$$43 - 20 \rightarrow 3$$

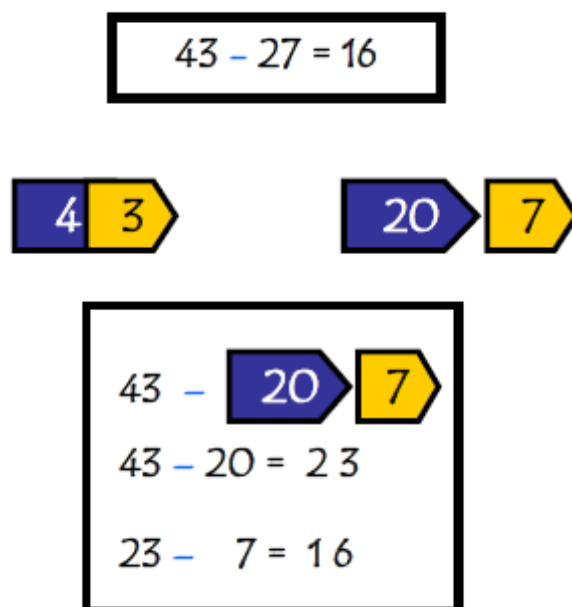
$$43 - 20 = 23$$

$$23 - 3 = 20$$

15. Decide whether to count on or count back

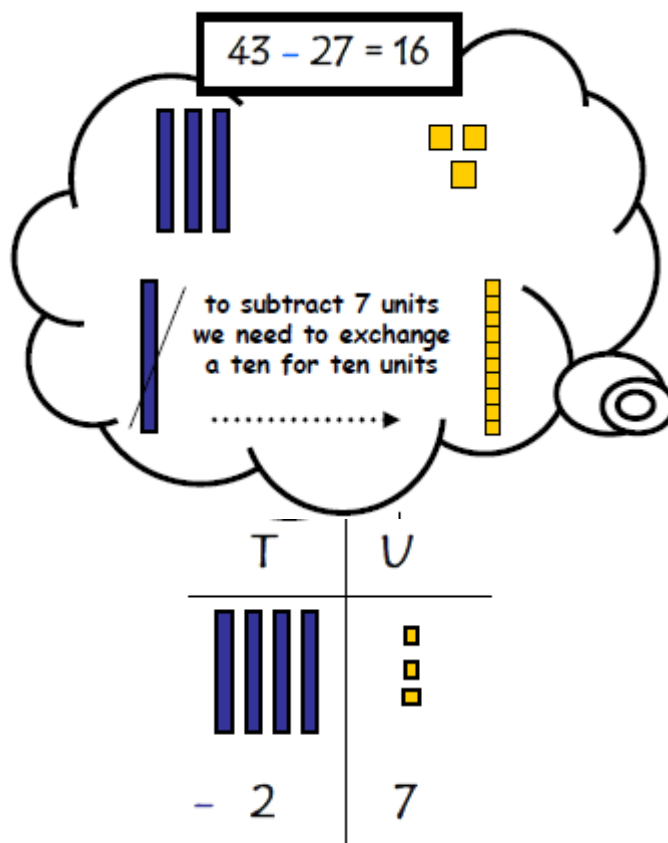


16. Partitioning number to be subtracted



17, Expanded method

It is important that the children have a good understanding of place value and partitioning using concrete resources and visual images to support calculations. The expanded method enables children to see what happens to numbers in the standard written method.



18. Standard written method

The previous stages reinforce what happens to numbers when they are subtracted using more formal written methods. It is important that the children have a good understanding of place value and partitioning.

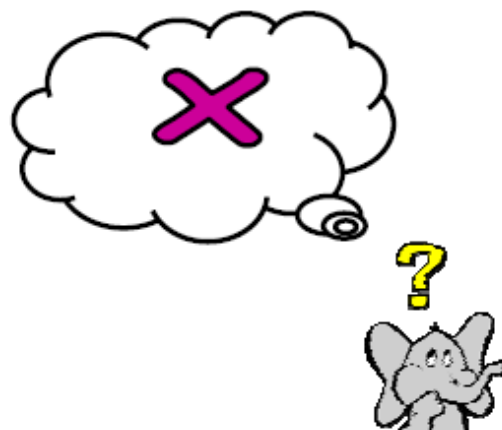
(Introduce 'exchanging' a ten)

$$\begin{array}{r} 3 \cancel{4} 3 \\ - 27 \\ \hline 16 \end{array}$$

Progression in Teaching Multiplication

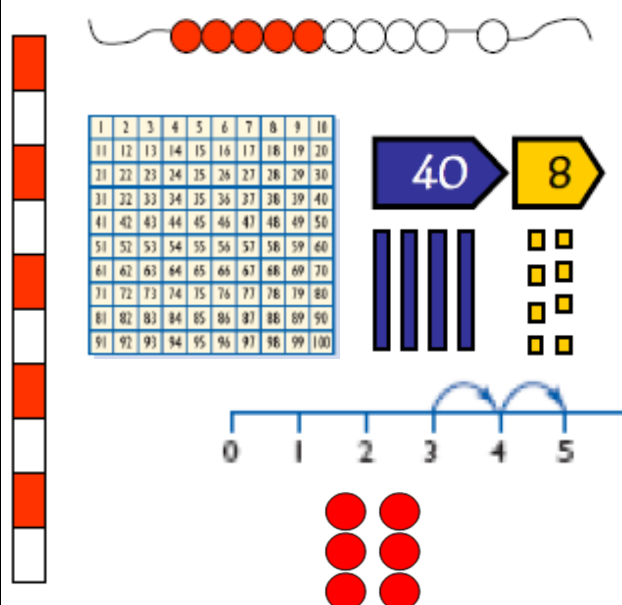
Mental Skills

Recognise the size and position of numbers
 Count on in different steps 2s, 5s, 10s and then all the other x tables
 Recognise multiplication as repeated addition
 Quick recall of multiplication facts
 Use known facts to derive associated facts
 Multiplying by 10, 100, 1000 and understanding the effect
 Multiplying by multiples of 10
 Know that the inverse of \times is \div check!



Models and Images

Counting apparatus
 Place value apparatus
 Arrays
 100 squares
 Number tracks
 Numbered number lines
 Marked but unnumbered lines
 Empty number lines
 Multiplication squares
 Counting stick
 Bead strings
 Models and Images charts
 ITPs–Multiplication grid, Number Dials,
 Multiplication Facts



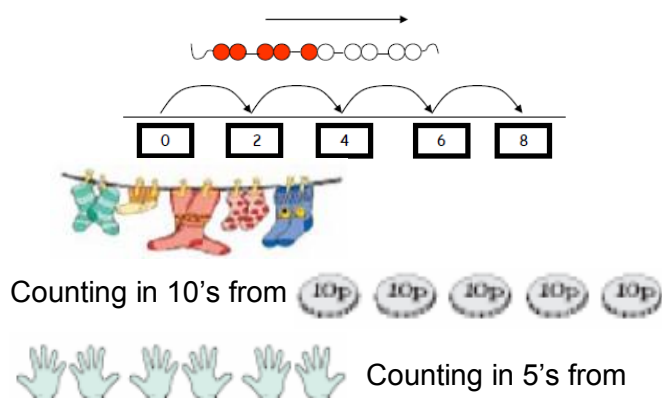
Vocabulary

Lots of
 groups of
 times
 Multiply
 multiplication
 multiple
 Product
 once, twice, three times
 array, row, column
 double
 repeated addition

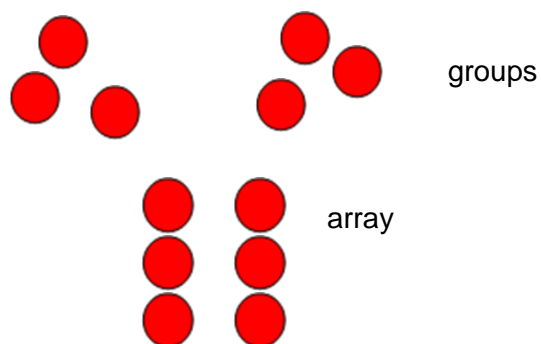
Written Multiplication

1. Counting practically in repeated groups.

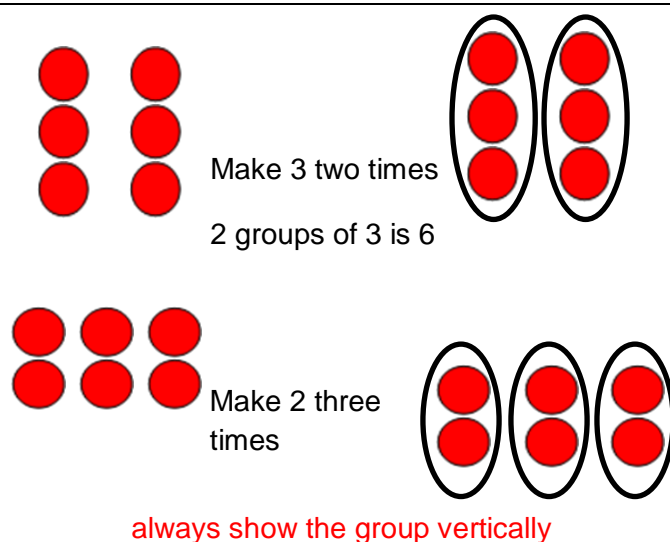
Children will experience equal groups of objects and will count in 2's, and 10's and begin to count in 5's. They will work on practical problem solving activities involving equal sets or groups.



2. Grouping and forming arrays to organise



3. Developing arrays.

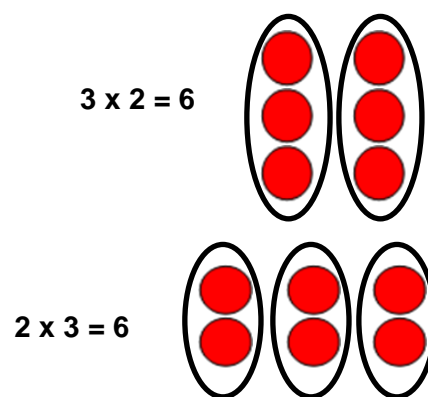


4. One digit multiplied by one digit in a number sentence

Eg. $3 \times 2 = 6$ $2 \times 3 = 6$

Use the same language as before when introducing the number sentence, ie for 2×3 "it's a group of two – three times. Reading backwards it becomes three lots of two."

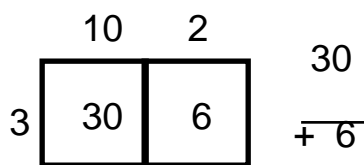
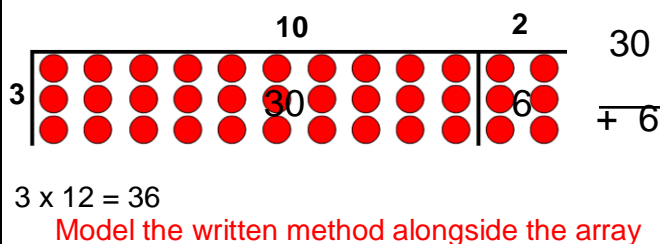
Show the number sentence alongside the array



5. Two digits multiplied by one digit introducing partitioning

Eg $3 \times 12 = 36$

This then becomes a numerical array:



6. Introduce short multiplication

14 start with units

x 6

~~24~~

7. Move to the more efficient method of short multiplication

(Then move to 3 digits by 1 digit, 4 digits x 1 digit etc.)

14 'exchange' 2 (20) sitting above the line

x 26

84

8. Long multiplication of two digits x two digits.

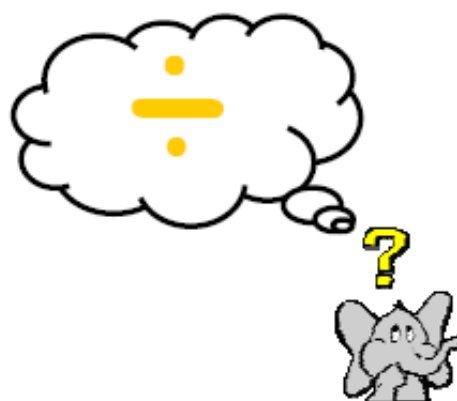
$$\begin{array}{r} 14 \\ \times 2 \\ \hline 28 \end{array}$$

start with units

Progression in Teaching Division

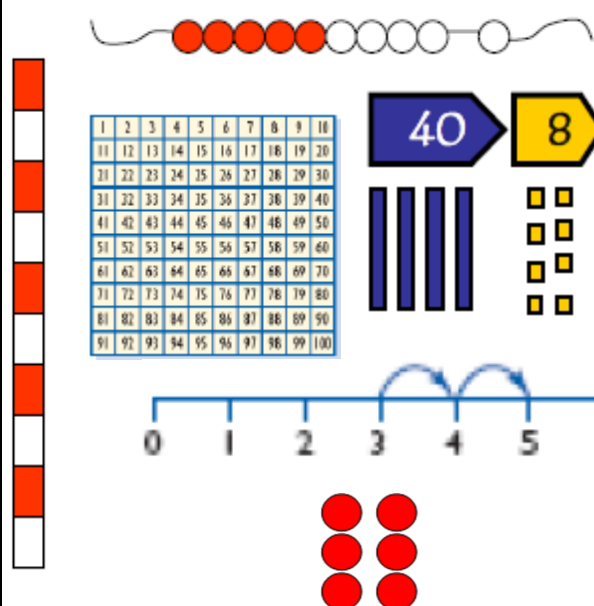
Mental Skills

Recognise the size and position of numbers
 Count back in different steps 2s, 5s, 10s, odd nos.
 Halve numbers to 20
 Recognise division as repeated subtraction
 Quick recall of division facts – use counting
 Use known facts to derive associated facts
 Divide by 10, 100, 1000 and understanding the effect
 Divide by multiples of 10
 Know that the inverse of \div is \times – check!



Models and Images

Counting apparatus
 Arrays
 100 squares
 Number tracks
 Numbered number lines
 Marked but unnumbered lines
 Empty number lines
 Multiplication squares
 Models and Images charts
 ITPs–Multiplication grid, Number Dials,
 Grouping
 Remainders



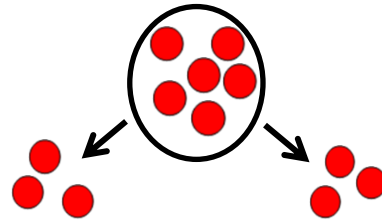
Vocabulary

lots of
 groups of
 share
 group
 halve
 half
 divide
 division
 divided by
 remainder
 factor
 quotient
 divisible

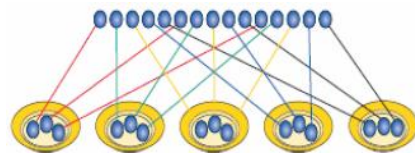
Written Division

1. Practical sharing.

Children will begin to understand equal groups and share items in practical ways using a range of equipment. They will begin to count in 2's and 10's and later in 5's.



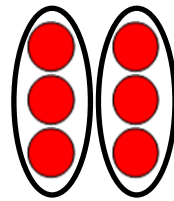
6 sweets shared between 2 people, how many do they get each?



15 shared between

2. Practical grouping

Groups are organised into an array

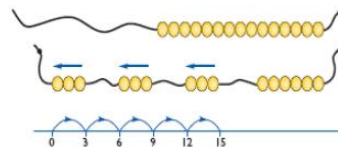


Always show groups vertically

How many groups of 3 can we make?

or

There are 6 sweets, how many people can have 2 sweets each?



How many 3s in 15?



$$15 \div 3 = 5$$

3. Introduce the ÷ sign by showing the array and the number sentence

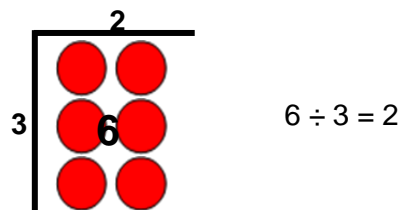
$$6 \div 3 = 2$$

*Be aware that this is the point where children can become confused by 'sharing' and 'making groups of' so it is important that the children hear and see this concept in a range of ways and hear a range of mathematical vocabulary: the vocabulary is dependent on the context at this point.

'divide'

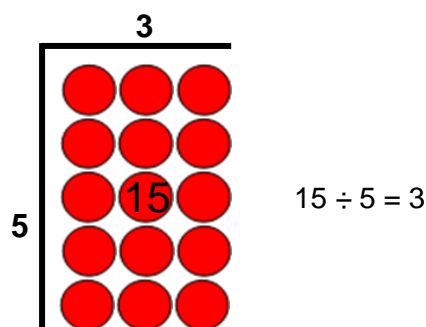
'share'

'groups of'



4. Increase to 10's 5's and 3's etc. using an array:

$$15 \div 5 = 3$$



5. Introduce short division

How many 5's are in 15?

$$\begin{array}{r} 3 \\ 5 \overline{) 15} \end{array}$$

There are 3 5's in 15

6. Develop short division

$$\begin{array}{r} 041 \\ 3 \overline{) 123} \end{array}$$

or

$$\begin{array}{r} 241 \\ 1 \overline{) 241} \end{array}$$

7. Develop further to dividing by a two digit number

$$\begin{array}{r} 021 \\ \overline{) 21} \end{array}$$

8. Divide by a two digit number with remainders

$$\begin{array}{r} 021 \text{ r}4 \\ 13 \overline{) 277} \end{array}$$